South Dakota School of Mines and Technology
Department of Materials and Metallurgical Engineering

MET 320 HQ 1 Oct 4, 2007

CLOSED BOOK and NOTES

- NO CALCULATORS
- Algebraic Answers Preferred
- Leave R in the equation but write out its value to achieve proper units.

UNITS (Algebraic answers should be left in a form to obtain these units

- q, w, U, and H [=] Joules
- S [=] J/K
- V[=]Liters
- T [=] K
- P [=] atm

SHOW ALL WORK ON THE SHEETS PROVIDED.
PLACE THE ANSWERS IN THE INDICATED BOX.

- Turn in only the problem sheets with the problems on them.
- Keep or discard all other paper

1. What is the configurational entropy change for 3 g-moles of Ar mixing with 7 g-moles of Ne?

$$\Delta S^{Mixing} = S^{M,ideal} \quad (Symbol\,Used\,in\,Later\,Chapers)$$

$$\Delta S^{Mixing} = -R\left[x_{Ne}\ln x_{Ne} + x_{Ar}\ln x_{Ar}\right]n_{Total}$$

$$S^{M,ideal} = -8.31\frac{J}{K*gmole}\left[\frac{7}{7+3}\ln\frac{7}{7+3} + \frac{3}{7+3}\ln\frac{3}{7+3}\right]10gmole$$

$$S^{M,ideal} = -8.31*10\left[0.7\ln 0.7 + 0.3\ln 0.3\right]\frac{J}{K}$$

$$S^{M,ideal} = 50.76\frac{J}{K}$$

2. **One** mole of ideal monotomic gas at 300 K is isothermally compressed from 1 atm to 10 atm. Find ΔU, ΔH, ΔS, q, w, and the final V.

$$T_2 = T_1 = 300K$$
$$P_2V_2 = P_1V_1$$
$$\Delta U = n * Cv * \Delta T = 0$$
$$\Delta H = n * Cp * \Delta T = 0$$
$$w_{Max} = q_{\text{Rev}} = \int_1^2 PdV = nRT\int_1^2 \frac{dV}{V} = nRT\int_1^2 d\ln V = nRT\ln\frac{V_2}{V_1} = nRT\ln\frac{P_1}{P_2} = 300R\ln\frac{1}{10} = -300R\ln 10$$
$$\Delta S = \int dS = \int \frac{dq}{T}\Big)_{\text{Rev}} = \frac{1}{T}\int dq\Big)_{\text{Rev}} = \frac{q_{\text{Rev}}}{T} = nR\ln\frac{P_1}{P_2} = -nR\ln 10 = -1gmole * 8.31\frac{J}{K * gmole}\ln 10$$
$$\Delta S = -19.13\frac{J}{K}$$
$$V_2 = \frac{nRT_2}{P_2} = \frac{R * 300K}{10atm} = \frac{1gmole * 0.08205\frac{L * atm}{K * gmole} * 300K}{10atm} = 2.46L$$

3. **Ten** moles of ideal gas at 300 K and 1 atm are adiabatically compressed to 10 atm. Find ΔU, ΔH, ΔS, q, w, and the final T.

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{R/Cp = R/(2.5R) = 0.4}$$
$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{0.4} = 300K\left(\frac{10}{1}\right)^{0.4} = 754K$$
$$\Delta T = (754 - 300)K = 454K$$
$$\Delta U = n * Cv * \Delta T = 10gmole * 1.5R * 454K = 10gmole * 1.5 * 8.31\frac{J}{gmole * K} * 454K = 56{,}540J$$
$$\Delta H = n * Cp * \Delta T = 10gmole * 2.5R * 454K = 10gmole * 2.5 * 8.31\frac{J}{gmole * K} * 454K = 94{,}230J$$
$$q_{\text{Rev}} = 0$$
$$\Delta S = \int dS = \int \frac{dq}{T}\Big)_{\text{Rev}} = 0$$

$$V_2 = \frac{nRT_2}{P_2} = \frac{R * 754K}{10atm} = \frac{1gmole * 0.08205\frac{L * atm}{K * gmole} * 754K}{10atm} = 6.19L$$

4. An ideal **diatomic** gas at 500 K and 1 atm is <u>isobarically</u> compressed from 10 liters to 1 liter. Find ΔU, ΔH, ΔS, q, w, n, and the final T.

$$Cp = 3.5R \qquad Cv = 2.5R$$

$$P_2 = P_1 = 1atm \qquad P = nR\frac{T_i}{V_i} \qquad V_i = nR\frac{T_i}{P}$$

$$\frac{T_2}{V_2} = \frac{T_1}{V_1}$$

$$n = \frac{P_1V_1}{RT_1} = \frac{1atm*10L}{0.08205\frac{L*atm}{K*gmole}*500K} = 0.24gmole$$

$$T_2 = T_1\frac{V_2}{V_1} = 500K*\frac{1L}{10L} = 50K$$

$$\Delta T = -450K$$

$$\Delta U = n*Cv*\Delta T = 0.24gmole*2.5R*(-450K)$$

$$= -0.24gmole*2.5*8.31\frac{J}{gmole*K}*450K = -2,244J$$

$$\Delta H = n*Cp*\Delta T = 0.24gmole*3.5R*(-450K)$$

$$= -0.24gmole*3.5*8.31\frac{J}{gmole*K}*450K = -3,141J$$

$$q_P = \Delta H = 3,141J$$

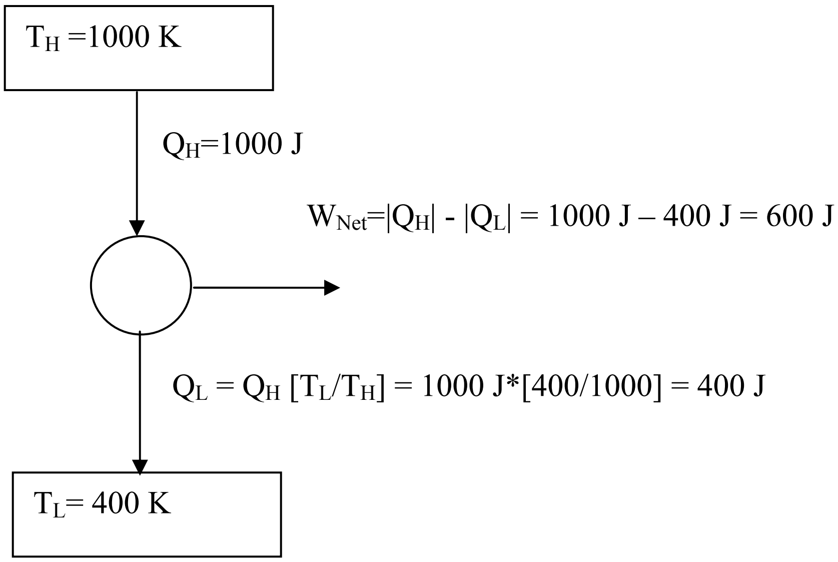
$$\Delta S = \int dS = \int \left.\frac{dq}{T}\right)_{Rev} = \int_1^2 \frac{nCpdT}{T} = n3.5R\int_1^2 d\ln T$$

$$= 0.24gmole*3.5*8.31\frac{J}{gmole*K}*\ln\frac{T_2}{T_1} = 6.98\frac{J}{K}*\ln\frac{50}{500} = -6.98\ln 10\frac{J}{K} = -16.07\frac{J}{K}$$

$$w_{Max} = \int_1^2 PdV = P\int_1^2 dV = P\left(V_2 - V_1\right) = nRP\left(\frac{T_2}{P} - \frac{T_1}{P}\right) = nR\left(T_2 - T_1\right)$$

$$= 0.24gmole*8.31\frac{J}{K*gmole}(-450K) = -897J$$

5a. A heat engine is operating between heat sinks at 1000 K and 400 K. What is the maximum theoretical work that can be produced from 1000 Joules of heat from the high temperature sink?



5b. A heat pump is operating between heat sinks at 27 degrees Celsius and 7 degrees Celsius. What is the theoretical work needed to deliver 1000 Joules of heat to the high temperature sink?