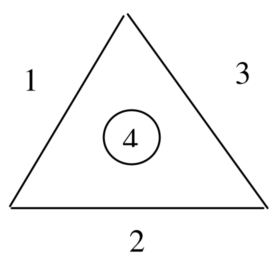
# South Dakota School of Mines and Technology
# Department of Materials and Metallurgical Engineering

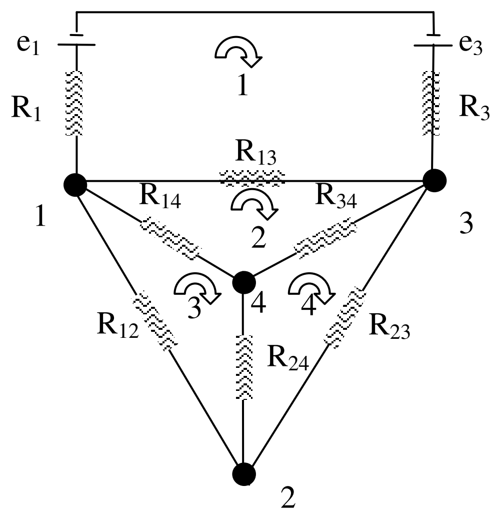
MET 422 | HQ 3 Solution Key | Dec 12, 2008

Open Book

**All needed physical constants are to be obtained from the text. Estimations are permissible if a needed constant is not available in the text.**

1. Draw the analogous electrical circuit and perform at least one Kirchhoff loop. Surfaces 2 and 4 are no net flux surfaces.




Loop 1: $0 = -R_3I_1 - R_{13}(I_1 - I_2) - R_1I_1 - e_1 + e_3$

Loop 2: $0 = -R_{13}(I_2 - I_1) - R_{34}(I_2 - I_4) - R_{14}(I_2 - I_3)$

Loop 3: $0 = -R_{14}(I_3 - I_2) - R_{24}(I_3 - I_4) - R_{12}I_3$

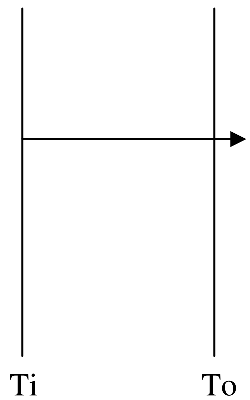
Loop 4: $0 = -R_{24}(I_4 - I_3) - R_{34}(I_4 - I_2) - R_{23}I_4$

Where

$$R_{ij} = \frac{1}{A_iF_{ij}}$$

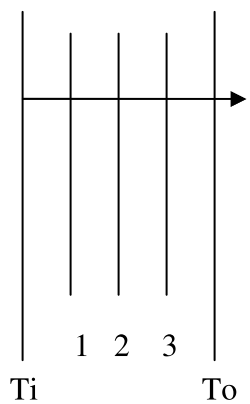
$$R_i = \frac{1-\varepsilon_i}{A_i\varepsilon_i}$$

2. Two black parallel plates are radiating 1000 Watts from one to the other. How many Watts would be exchanged if three heat shields were place between them with surface emissivity of 0.2?




Given:

$$Q=\frac{\sigma\left(T_i^4-T_0^4\right)}{\left(\frac{1}{\varepsilon_i}+\frac{1}{\varepsilon_0}-1\right)}=\frac{\sigma\left(T_i^4-T_0^4\right)}{\left(\frac{1}{1}+\frac{1}{1}-1\right)}=\sigma\left(T_i^4-T_0^4\right)=1000\text{ W}$$




Solution:

$$Q=\frac{\sigma\left(T_i^4-T_0^4\right)}{\left(\frac{1}{\varepsilon_i}+\frac{1}{\varepsilon_1}-1\right)+\left(\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_2}-1\right)+\left(\frac{1}{\varepsilon_2}+\frac{1}{\varepsilon_3}-1\right)+\left(\frac{1}{\varepsilon_3}+\frac{1}{\varepsilon_0}-1\right)}$$

$$Q=\frac{1000\text{ W}}{\left(\frac{1}{1}+\frac{1}{0.2}-1\right)+\left(\frac{1}{0.2}+\frac{1}{0.2}-1\right)+\left(\frac{1}{0.2}+\frac{1}{0.2}-1\right)+\left(\frac{1}{0.2}+\frac{1}{1}-1\right)}$$

$$Q=\frac{1000\text{ W}}{5+9+9+5}$$

$$\boxed{Q=\frac{1000\text{ W}}{28}}$$

3. a) Derive the concentration gradient for water vapor in a Tygon® polymer tube wall with an inside radius of 1 cm and a wall thickness of 2 cm. The inside vapor pressure of the water in the dry $N_2$ inside the tube is 0 while the pressure in the atmosphere surrounding the tubing is Po.
   b) What is the rate at which water vapor diffuses into the dry $N_2$ per meter of tubing?

a) For SS Cyl coordinates with no Generation:

$$In - Out + Gen = Acc$$

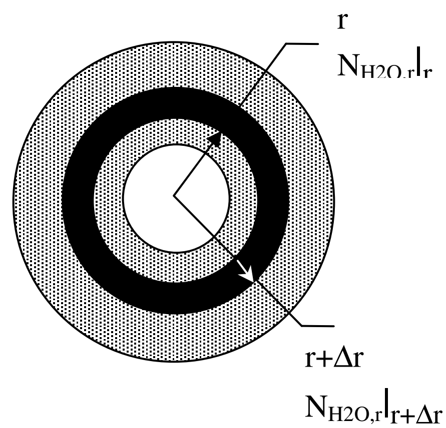
$$2\pi L\left[\left(rN_{H_2O,r}\right)\Big|_r - \left(rN_{H_2O,r}\right)\Big|_{r+\Delta r}\right] + 0 = 0$$

$$\frac{\partial (rN_{H_2O,r})}{\partial r} = 0$$

$$rN_{H_2O,r} = C_1$$

$$-\mathscr{D}_{H_2O-Tygon}\frac{\partial C_{H_2O}}{\partial r} = \frac{C_1}{r}$$

$$C_{H_2O} = \frac{C_1}{-\mathscr{D}_{H_2O-Ar}}\ln(r) + C_2$$




Apply BC's:

#1 $C=C_i$ @ $r = R_i$ #2 $C=C_o$ @ $r = R_o$

$$C_i = \frac{C_1}{-\mathscr{D}_{H_2O-Tygon}}\ln(R_i) + C_2 \qquad C_o = \frac{C_1}{-\mathscr{D}_{H_2O-Tygon}}\ln(R_o) + C_2$$

$$C_1 = \frac{\mathscr{D}_{H_2O-Tygon}\left(C_i - C_o\right)}{\ln\left(R_o/R_i\right)} \qquad C_2 = \frac{C_1}{\mathscr{D}_{H_2O-Tygon}}\ln(R_o) + C_o$$

$$C_{H_2O} = C_1\left[\frac{1}{-\mathscr{D}_{H_2O-Tygon}}\ln(r) + \frac{1}{\mathscr{D}_{H_2O-Tygon}}\ln(R_o)\right] + C_o$$

$$\left(C_{H_2O} - C_o\right) = C_1\left[\frac{1}{\mathscr{D}_{H_2O-Tygon}}\ln\left(\frac{R_o}{r}\right)\right]$$

$$\left(C_{H_2O} - C_o\right) = \frac{\mathscr{D}_{H2O-Tygon}\left(C_i - C_o\right)}{\ln\left(R_o/R_i\right)}\left[\frac{1}{\mathscr{D}_{H_2O-Tygon}}\ln\left(\frac{R_o}{r}\right)\right]$$

$$\frac{\left(C_{H_2O} - C_o\right)}{\left(C_i - C_o\right)} = \ln\left(r/R_o\right)/\ln\left(R_i/R_o\right)$$

$$\boxed{1 - C_{H_2O}/C_o = 1 - \ln\left(r\right)/\ln\left(3\right)}$$

b) The total rate

$$\mathscr{W} = 2\pi r L N_{H2O,r} = 2\pi r L\frac{C_1}{r} = 2\pi L C_o\frac{\mathscr{D}_{H2O-Tygon}}{\ln\left(3\right)}$$

$$\boxed{\mathscr{W}/L = 2\pi\frac{P_o}{RT}\frac{\mathscr{D}_{H2O-Tygon}}{\ln\left(3\right)}}$$

4. A power line wire is coated with ice to a diameter of 2 cm. The dry air at -13 °C from Alaska is blowing over the wire at 22.4 MPH (10 m/s). Calculate the mass rate loss by sublimation from the wire per meter of length. The vapor pressure of ice at -13 °C is 5.7x10$^{-8}$ atm.

$$\mathcal{W} = \pi D L N_{H2O,r}$$

$$N_{H2O,r} = k_m \left( C_i - C_o \right) = k_m \left( \frac{P_i}{RT} - 0 \right)$$

$$k_m = Nu_m D / \mathcal{D}_{H20-Ar}$$

$$Nu_m = f(\mathrm{Re}, Sc) \qquad \text{Use Fig 8.21 for Nu (HT)}$$

$$\mathrm{Re} = \frac{0.02m * 10m/s * 1.2Kg/m^3}{1.6x10^{-5} Kg/s/m} = 15000$$

$$Sc = \frac{\eta}{\rho \mathcal{D}_{H20-Ar}} = \frac{1.6x10^{-5} Kg/s/m}{1.2Kg/m^3 * 10^{-5} m^2/s} = 1.3$$

Fig 8.5 is a plot of

$$j_H = \frac{h}{\rho C p V} \mathrm{Pr}^{2/3} = h \frac{D}{D} \frac{k}{k} \frac{\eta}{\eta} \frac{1}{\rho C p V} \mathrm{Pr}^{2/3}$$

$$= \frac{hD}{k} \frac{\eta}{DV\rho} \frac{k}{\eta C p} \mathrm{Pr}^{2/3} = \frac{Nu}{\mathrm{Re}} \frac{\mathrm{Pr}^{2/3}}{\mathrm{Pr}} =$$

$$j_H = \frac{Nu}{\mathrm{Re}\,\mathrm{Pr}^{1/3}} \qquad \text{Eq. [8.6]}$$

The MT equivalent of this is

$$J_M = \frac{Nu_m}{\mathrm{Re}\, Sc^{1/3}} \qquad \text{Eq. [14.80]}$$

$j_H = 0.005$. Figure 8.5. Therefore,

$$Nu_m = 0.005 * \mathrm{Re}\, Sc^{1/3} = 0.005 * 15000 * 1.3^{1/3} = 82$$

$$k_m = 82 * \mathcal{D}_{H2o-Ar} / D = 0.041 m/s$$

$$N_{H2O,r} = k_m \left( \frac{5.7x10^{-8} atm}{0.08205L * atm/K/gmole * 260K} \frac{1000L}{m^3} \right) = 1.09x10^{-7}\, gmoles/s/m^2$$

$$\mathcal{W}_{H2O} = 0.02\pi m^2 * 1.09x10^{-7}\, gmoles/s/m^2 = 0.069x10^{-6}\, gmoles/s = 0.106 \text{ g/day}$$

$\boxed{\mathcal{W}_{H2O} = 1.68\ \mu\text{m/day @ -13 °C}}$

Compare with $\left[ \mathcal{W}_{H2O} \approx 1.78 \text{ mm/day @ } 0\ \text{°C} \right]$