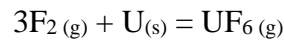


Derive the concentration profile for $F_{2(g)}$ reacting with a U sphere to produce $UF_{6(g)}$ by the reaction



Assume the U is intermingled with fine particles of Al_2O_3 that form a porous layer of alumina through which the above gases diffuse. Use the following BC's: at $r=r_0$, $X_{F_2}=X^0$ and at $r=r_1$, $X_{F_2}=1$.

Solution

First determine the form of Fick's Law including the bulk diffusion terms.

$$N_{F_2} = X_{F_2} (N_{F_2} + N_{UF_6}) - C \mathcal{D}_{F_2,UF_6} \frac{\partial X_{F_2}}{\partial r} \quad \text{text Eq. (14.4)}$$

$$N_{F_2} = -3N_{UF_6} \quad (1)$$

$$N_{F_2} = -\frac{C \mathcal{D}_{F_2,UF_6}}{\left(1 - \frac{2}{3} X_{F_2}\right)} \frac{\partial X_{F_2}}{\partial r} \quad (2)$$

$$N_{F_2} = \frac{3C \mathcal{D}_{F_2,UF_6}}{2} \frac{\partial \ln\left(1 - \frac{2}{3} X_{F_2}\right)}{\partial r} \quad (3)$$

Second, write the known relationship for the change of a flux in spherical coordinates at steady state.

$$\frac{\partial(r^2 N_{F_2})}{\partial r} = 0 \quad (4)$$

Substitute in Fick's Law

$$\frac{\partial \left(r^2 \frac{3C \mathcal{D}_{F_2,UF_6}}{2} \frac{\partial \ln\left(1 - \frac{2}{3} X_{F_2}\right)}{\partial r} \right)}{\partial r} = 0 \quad (5)$$

Rearrange and integrate once, indefinitely.

$$\left(\frac{r^2 \partial \ln\left(1 - \frac{2}{3} X_{F_2}\right)}{\partial r} \right) = C_1 \quad (6)$$

Integrate again to obtain

$$\ln\left(1 - \frac{2}{3} X_{F_2}\right) = -\frac{C_1}{r} + C_2 \quad (7)$$

Apply the BC's.

$$\ln\left(1 - \frac{2}{3} X_{F_2}^\circ\right) = -\frac{C_1}{r_o} + C_2 \quad (8)$$

$$\ln\left(\frac{1}{3}\right) = -\frac{C_1}{r_1} + C_2 \quad (9)$$

Solve for C_1 and C_2 .

$$C_1 = \frac{\ln(3 - 2X_{F_2}^\circ)}{\left[\frac{1}{r_1} - \frac{1}{r_o}\right]} \quad (10)$$

$$C_2 = -\frac{C_1}{r_1} - \ln\left(\frac{1}{3}\right) \quad (11)$$

Substituting into Eq (8) gives

$$\ln\left(1 - \frac{2}{3} X_{F_2}\right) = -\frac{\ln(3 - 2X_{F_2}^\circ)}{\left[\frac{1}{r_1} - \frac{1}{r_o}\right]} \left(\frac{1}{r} - \frac{1}{r_1}\right) + \ln\left(\frac{1}{3}\right), \quad (12)$$

which may be written in dimensionless form as

$$\frac{\ln(3 - 2X_{F_2})}{\ln(3 - 2X_{F_2}^\circ)} = -\frac{\left[\frac{1}{r} - \frac{1}{r_1}\right]}{\left[\frac{1}{r_1} - \frac{1}{r_o}\right]}. \quad (13)$$