Elementary PDQs

Oct 15, 1999

1. a) Write the Heat Equation in incremental form and
   b) solve it for the new temperature.

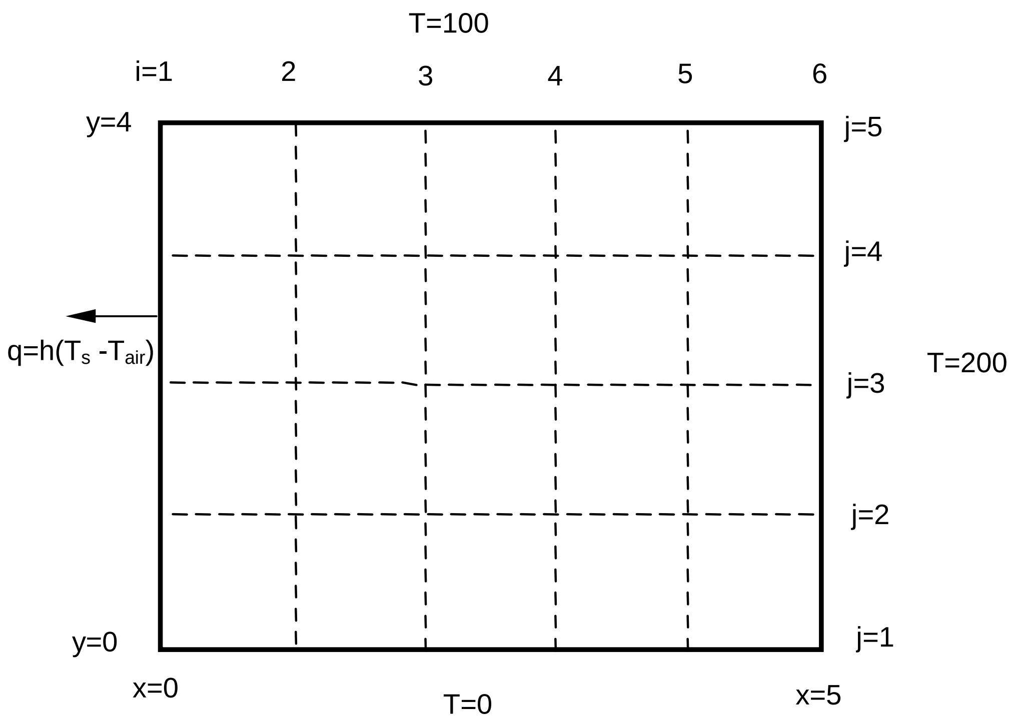
$$\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)=\frac{\partial T}{\partial t}$$

2. Sketch a spreadsheet solution to determine temperature profiles in a one-dimensional system as a function of position and time. Assume the solid of interest is 10 cm thick. Use 10 increments. The thermal diffusivity is 0.10 cm^2/sec. Use the maximum permissible time step. Show
   a) all pertinent equations
   b) boundary conditions,
   c) Initial conditions, and
   d) the value of the time step.

3. Given the data below, what is the largest time step allowed in the simple explicit method of solving a 1D USS HT problem.

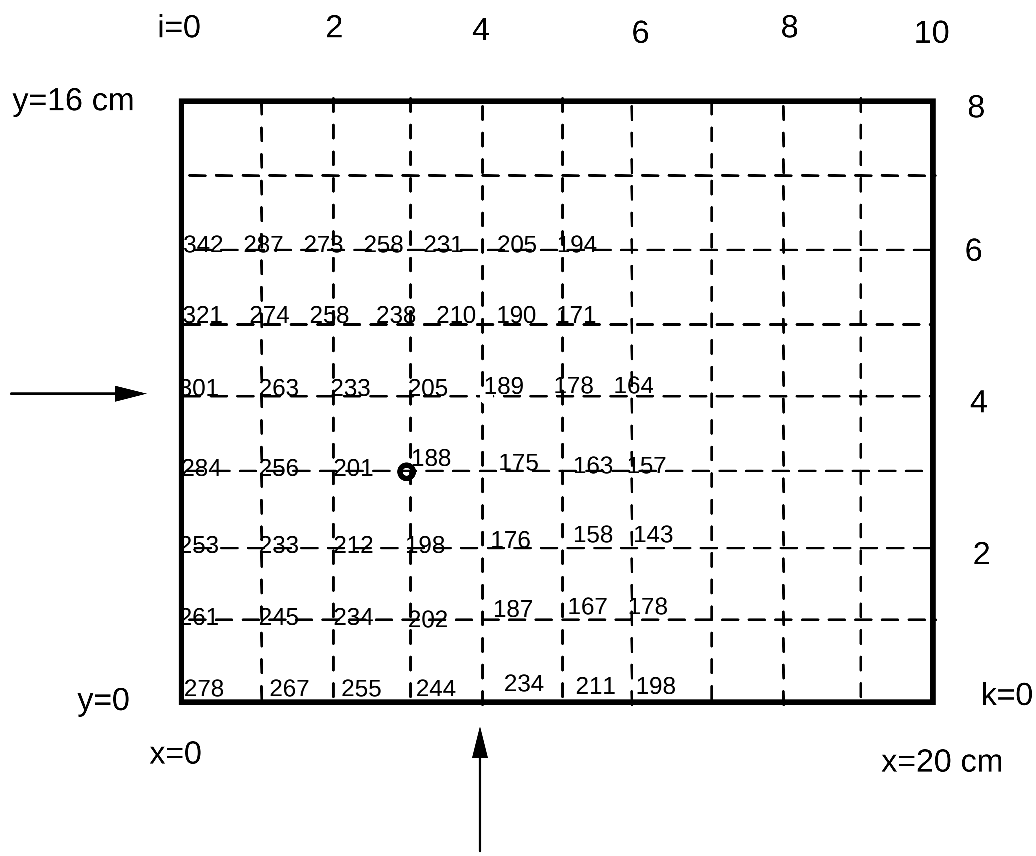
   alpha = 0.4 cm^2/sec
   $\Delta x = 0.2$ cm

5. The steady-state temperature profile for the plate below is desired. There is a convection boundary condition on the left side as shown. Write enough equations to show how to solve for the temperature profile.



Final 1999S

4. Below is a temperature profile at time = 0. Find the temperature at i=3 and j=3 at time = 20 seconds. Assume the largest time step consistent with a stable solution. The value of α is 0.10 cm^2/sec. All dimensions are in cm. and all temperatures are in °C. State any assumptions made.



NOTE: $\frac{1}{\alpha}\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}$

5. Write the **forward and central** derivative in incremental form for $\frac{\partial T}{\partial t}$.

Hour Exam 2000S

Place your answers on the answer sheet provided. Use a #2 pencil. Ink will NOT be detected by the scoring machine.

1. Which of the following is NOT one of the five steps to deriving a differential equation?
   A. Substitute the flux equation (i.e. Fourier's Law, Newton's Law of Viscosity, etc.)
   B. Make a sketch
   C. Divide by the independent Δ's and take the limit as they go to zero
   D. Use the BC's to evaluate C_1 and C_2

2. What is the shape of a differential element for a three-dimensional heat conduction problem in rectilinear coordinates (3D USS HT)?
   A. An infinite flat sheet Δx thick
   B. An infinitely long French-fry-shaped element with a cross section Δy by Δx
   C. A small cube Δx by Δy by Δz
   D. None of the above

3. What is the shape of a differential element for a heat conduction problem in a cylinder in which the temperature varies in both the radial and axial directions? No generation.
   A. A solid cylinder L long with
   B. A tube L long with a radius r and a wall thickness of Δr
   C. A ring Δz long with a radius r and wall thickness of Δr
   D. A thin disk Δz thick with a radius r

4. What shape of the differential element for a heat conduction problem involving a sphere in which the temperature varies with r?
   A. A solid sphere with radius Δr
   B. A solid sphere with radius r
   C. A spherical shell Δr thick with radius r
   D. A small element Δθ, dϕsinθ, by Δr at radius r.

5. What is the area through which the radial flux (r-dir) moves in a cylindrical element L long?
   A. πrL
   B. 2πrL
   C. πr^2
   D. 2πrΔr

6. What is the area through which an axial flux (z-dir) moves through a cylindrical element Δz.
   A. πr^2
   B. 2πrΔz
   C. 2πrΔr
   D. 2πrL

7. What is the volume of a spherical differential in which r is the only independent position variable?.
   A. πr^3
   B. $4\pi r^2\Delta r$
   C. 2πrLΔz
   D. $(4/3)\pi r^2\Delta r$

8. Which is the correct equation for a heat balance for a cylindrical coordinate heat conduction problem in which temperature varies in the radial direction only? No generation.
   A. $\left[(2\pi L r q_r)|_r - (2\pi L r q_r)|_{r+\Delta r}\right]\Delta t = 2\pi L r \Delta r \rho C p\left(T_{t+\Delta t} - T_t\right)$
   B. $2\pi L r\left[(q_r)|_r - (q_r)|_{r+\Delta r}\right]\Delta t = 2\pi L r \Delta r \rho C p\left(T_{t+\Delta t} - T_t\right)$
   C. $2\pi L\left[(r q_r)|_r - (r q_r)|_{r+\Delta r}\right]\Delta t = \pi r^2 L \rho C p\left(T_{t+\Delta t} - T_t\right)$
   D. None of the above

9. Which is the correct equation for a rectilinear coordinate heat conduction problem in which temperature varies in the x and y directions only? No generation. The solid is W wide (x-dir), H high (y-dir), and L long (z-dir).
   A. $\left[L\Delta y\left(q_x|_x - q_x|_{x+\Delta x}\right) + L\Delta x\left(q_y|_y - q_y|_{y+\Delta y}\right)\right]\Delta t = L\Delta x \Delta y \rho C p\left(T_{t+\Delta t} - T_t\right)$
   B. $\left[W\Delta x\left(q_x|_x - q_x|_{x+\Delta x}\right) + W\Delta y\left(q_y|_y - q_y|_{y+\Delta y}\right)\right]\Delta t = W\Delta x \Delta y \rho C p\left(T_{t+\Delta t} - T_t\right)$
   C. $\left[\Delta y\left(q_x|_x - q_x|_{x+\Delta x}\right) + \Delta x\left(q_y|_y - q_y|_{y+\Delta y}\right)\right]\Delta t = \Delta x \Delta y L \rho C p\left(T_{t+\Delta t} - T_t\right)$
   D. None of the above

10. What is the definition of the derivative $\frac{dy}{dx}$?.

A. $Lim\ as\ x \to 0\ \frac{y|_{x+\Delta x} - y|_x}{\Delta x}$

B. $Lim\, as\, x \to 0 \frac{y|_x - y|_{x+\Delta x}}{\Delta x}$

C. $Lim\, as\, x \to 0 \frac{y|_{x+\Delta x} - y|_x}{\Delta y}$

D. None of the above

Correct Answers: 1-D 2-C 3-C 4-C 5-B 6-C 7-B 8-A 9-A 10-D

Adjustments: #6 could be A since it was not specified that there was any change in the r direction
#8 20% credit will be given for B if you promise to never make that mistake again
#10 50% credit will be given for A since it is correct except for no delta in the limit

Hour Exam 2001F

2 Sketch a spreadsheet solution to determine temperature profiles in a one-dimensional rectilinear solid as a function of position and time (1D USS HT). The solid's initial temperature is 0 and the end temperatures are both 100 for t ≥ 0. Assume the solid of interest is 20 cm thick. Use 10 increments. The thermal diffusivity is 0.125 cm^2/sec. Use the maximum permissible time step. Show

a) Δx,
b) Δt,
c) initial condition,
d) the boundary conditions, and
e) the pertinent equations for computation. You may indicate "Fills" to conserve effort writing.

<u>Answer: (active Excel Spreadsheet)</u>

	x										
time	0	2	4	6	8	10	12	14	16	18	20
0	100	0	0	0	0	0	0	0	0	0	100
16	100	50	0	0	0	0	0	0	0	50	100
32	100	50	25	0	0	0	0	0	25	50	100
48	100	63	25	13	0	0	0	13	25	63	100
64	100	63	38	13	6	0	6	13	38	63	100
80	100	69	38	22	6	6	6	22	38	69	100
96	100	69	45	22	14	6	14	22	45	69	100
112	100	73	45	30	14	14	14	30	45	73	100
128	100	73	51	30	22	14	22	30	51	73	100
144	100	76	51	37	22	22	22	37	51	76	100

3. Write the finite difference approximation for the following:

BONUS

a) Forward $\frac{\partial T}{\partial t} \approx \frac{T_{t+\Delta t} - T_t}{\Delta t}$ O(h^N), N=___1__

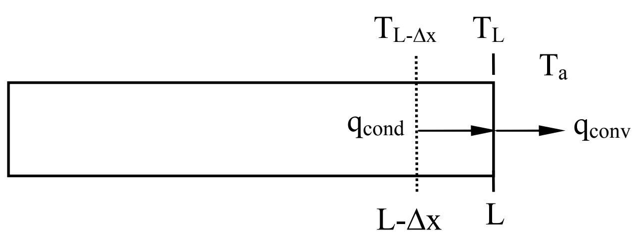
b) Central $\frac{\partial T}{\partial t} \approx \frac{T_{t+\Delta t} - T_{t-\Delta t}}{2\Delta t}$ O(h^N), N=___2__

c) Backward $\frac{\partial T}{\partial t} \approx \frac{T_t - T_{t-\Delta t}}{\Delta t}$ O(h^N), N=___1__

d) Central $\frac{\partial^2 T}{\partial x^2} \approx \frac{T_{x-\Delta x} - 2T_x - T_{x+\Delta x}}{\Delta x^2}$ O(h^N), N=___2__

5. The solid bar below is conducting along its axis while heat is being lost by convection from the ends. Derive an equation showing the temperature T_L as a function of $T_{L-\Delta x}$, and T_a.

Note: $q_{conv} = h(T_L - T_a)$ and $q_{cond} = -k(dT/dx)$



Answer:

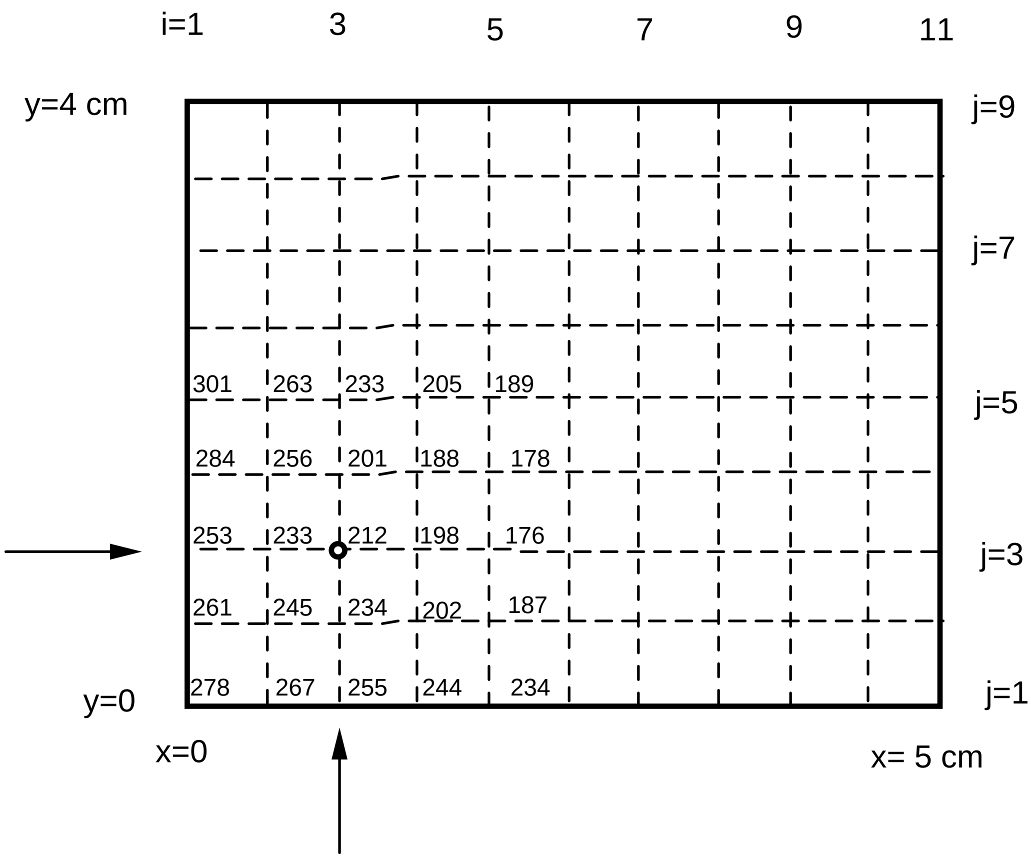
$$q_{Cond} = q_{Conv}$$

$$-k\frac{T_L - T_{L-\Delta x}}{\Delta x} = h(T_L - T_a)$$

$$T_L = \frac{hT_a + \frac{k}{\Delta x}T_{L-\Delta x}}{h + \frac{k}{\Delta x}}$$

Hour Exam 1998F

7. (15) Below is a temperature profile in a plate at a particular time. Assume the largest time step consistent with a stable explicit solution. Find the temperature at i=3 and j=3 after one additional time step.



i=1
3
5
7
9
11
y=4 cm
j=9
j=7
j=5
j=3
j=1
301
263
233
205
189
284
256
201
188
178
253
233
212
198
176
261
245
234
202
187
278
267
255
244
234
y=0
x=0
x= 5 cm

Hour Exam 1998F

1. (15) a) Write the **Heat Equation** in incremental form and
   b) solve it for the new temperature. when $f = k_2$

$$\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)=\frac{\partial T}{\partial t}$$

a) $\alpha\left(\frac{T_{+x}-2T+T_{-y}}{\Delta x^2}+\frac{T_{+y}-2T+T_{-y}}{\Delta y^2}\right)=\frac{T'-T}{\Delta t}$

b) $T' = (T_{+x}+T_{-x}+T_{+y}+T_{-y})/4$

2. (10) Write the a) forward
   b) central

   derivative in incremental form for $\frac{\partial T}{\partial t}$.

a) $\frac{T'-T}{\Delta t}$

b) $\frac{T'-{}'T}{2\Delta t}$

3. (15) Given the data below, what is the largest time step allowed in the simple explicit method of solving a 1D USS HT problem.

   alpha = 0.4 cm^2/sec
   Δx = 0.2 cm

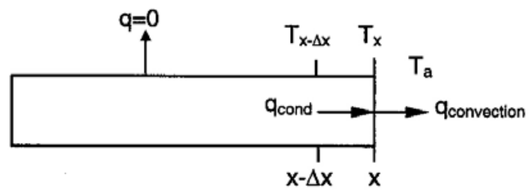
$$\frac{\alpha\Delta t}{\Delta x^2}=\frac{1}{2}$$

$$\Delta t=\frac{1}{2}\Delta x^2/\alpha=\frac{1}{2}\,\frac{0.2^2}{0.4}\ \text{sec}$$

4. (10) The solid bar below is conducting heat along its horizontal axis while heat is being lost by convection from the end as shown. Derive the boundary equation showing the temperature T_x as a function of $T_{x-\Delta x}$, and T_a. (Assume there is no heat loss from the surface perpendicular to the horizontal axis of the bar.)

Note: $q_{conv} = h(T-T_a)$

$q_{cond} = -k(dT/dx)$

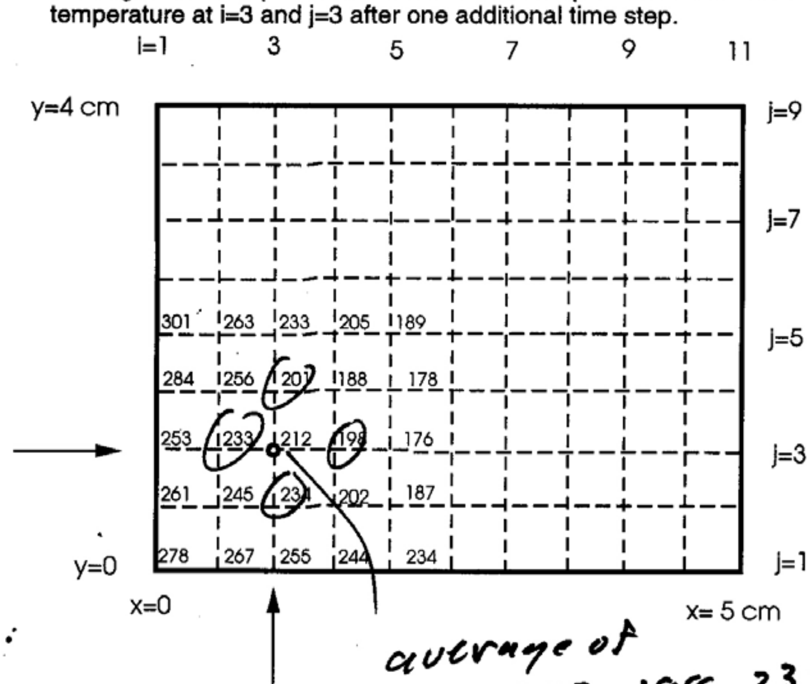


$q_{cond} = q_{conv}$

$$-k\,\frac{T_x - T_{x-\Delta x}}{\Delta x} = h\,(T_x - T_a)$$

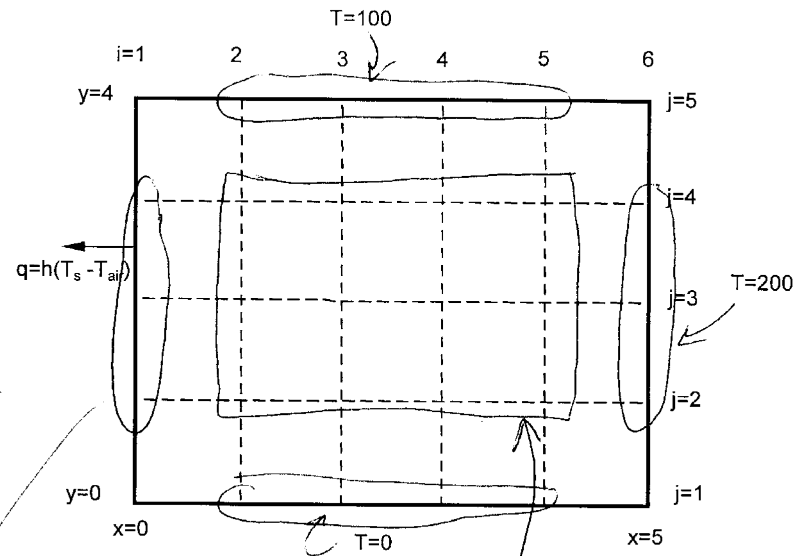
solve for T_x

7. (15) Below is a temperature profile in a plate at a particular time. Assume the largest time step consistent with a stable explicit solution. Find the temperature at i=3 and j=3 after one additional time step.



average of
201, 233, 198, 234

5. The steady state temperature profile for the plate below is desired. There is a convection boundary condition on the left side as shown. Write enough equations to show how to solve for the temperature profile.



$$T_{i,j} = \frac{T_{i+1,j} + T_{i-1,j} + T_{i,j+1} + T_{i,j-1}}{4}$$

$q_{cond} = q_{conv}$ $\quad i = 1$

$$-k\frac{T_{i+1,j} - T_{i,j}}{\Delta x} = h(T_{i,j} - T_{air})$$

$$-k\frac{(T_{2,j} - T_{1,j})}{\Delta x} = h(T_{1,j} - T_{air})$$

$$T_{1,j} = \frac{h T_{air} - k/\Delta x \, T_{2,j}}{h - k/\Delta x} \quad \text{OR} \quad T_S = \frac{h T_{air} - \frac{k}{\Delta x} T_{S+1}}{h - k/\Delta x}$$

Hour Exam 2000S

3. Sketch a spreadsheet solution to determine temperature profiles in a two-dimensional system at steady state. Assume the solid of interest is 100 cm wide and 200 cm long with no temperature gradients in the thickness direction.. Use ten increments in the 100-cm direction.

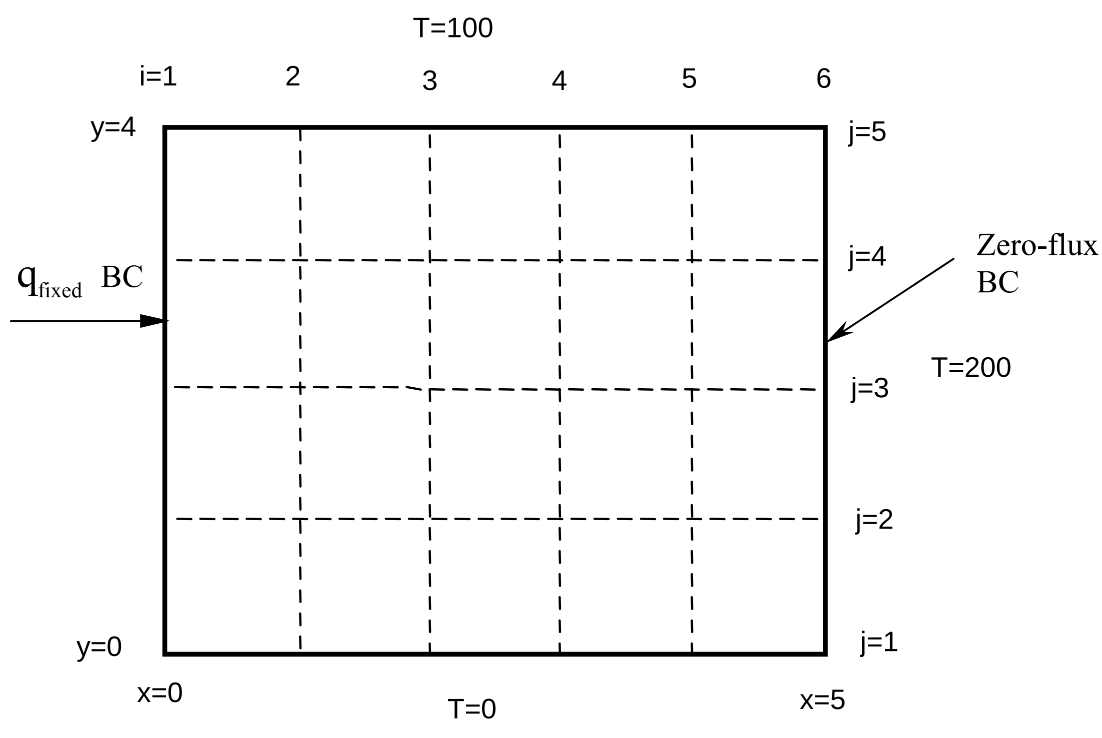
   alpha = 0.04 cm^2/sec

   Show
   a) The size and number of increments in the length direction.
   b) All pertinent equations
   c) Boundary conditions,
   d) Initial conditions, and

4. Given the data below, what is the largest time step allowed in the simple explicit method of solving a 2D USS HT problem.

   alpha = 0.05 cm^2/sec
   Δx = 10 cm

5. The steady state temperature profile for the plate below is desired (2D SS HT). There is a fixed (known) flux boundary condition on the left side, a zero-flux boundary condition on the right side, and fixed temperature boundary conditions on the top and bottom as shown. Derive the equations (in algebraic form not spreadsheet format) that would be needed for the spreadsheet cells for the right and left boundary conditions equations to show how to solve for the temperature profile.



Please put you solution on the following page.

Final 2001F

1. Incremenation of partial differentials:
   a) Write the following partial derivative in incremental form and state on the line provided whether your response is a backward, central, or forward approximation.

   ____________ $\frac{\partial T}{\partial t} =$

   b) Provide an explicit solution for the new T resulting from putting the Heat Equation into incremental form.

   $$\alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) = \frac{\partial T}{\partial t}$$

6. Given the data below, what is the largest time step allowed in the simple explicit method of solving the Heat Equation when alpha = 0.25 cm^2/sec and $\Delta x = \Delta y = \Delta z = 0.1$ cm in
   a) One dimension
   b) Two dimensions
   c) Three dimensions

Final 2002F

1. a) Write the 2D USS Heat Equation in incremental form.

   $$\alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) = \frac{\partial T}{\partial t}$$

   b) Solve it for the new temperature at any time step.
   c) Show the solution for the maximum time step.

4. Derive the 1D USS HT equation in rectilinear coordinates. Include a generation term, S, per unit volume. Show your work in detail.

5. Solve the equation $\alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) = \frac{\partial T}{\partial t}$ for the steady state temperature as needed in the spreadsheet solution

6. Given the data below, what is the largest time step allowed in the method of solving a 1D USS HT problem by the methods covered so far in class if
   a) $\alpha = 0.5$ cm^2/sec and $\Delta x = 0.2$ cm?
   b) k = 1.0 J/(cm*K*sec),
      Cp = 0.5 J/(g*K), and
      ρ = 8 g/cm^3 ?

Final 2005S

2. The equation for a vibrating string is $\frac{\partial^2 y}{\partial t^2} = a\frac{\partial^2 y}{\partial^2 x}$.

   a) How many boundary conditions are needed to solve this equation?
   c) How many conditions in time are needed to solve this equation?
   c) Write an approximation of the equation in incremental form.

3. A question concerning the 1D USS HC equation with no generation.
   a) Derive the equation in rectilinear coordinates.
   b) Write the equation in cylindrical and spherical coordinates

Final 2005F

2. Incremenation of partial differentials:
   a) Write the following partial derivative in incremental form and state on the line provided whether your response is a backward, central, or forward approximation.

      ____________ $\frac{\partial T}{\partial t} =$

   b) Provide an explicit solution for the new T resulting from putting the Heat Equation into incremental form.

      $$\alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) = \frac{\partial T}{\partial t}$$

2. A question concerning the 1D USS HC Equation with no generation.
   a) Derive the equation in rectilinear coordinates.
   b) Write (or derive, if need be) the equation in cylindrical and spherical coordinates.

3. The equation for a vibrating string is $\frac{\partial^2 y}{\partial t^2} = a\frac{\partial^2 y}{\partial^2 x}$.
   a) How many boundary conditions are needed to solve this equation?
   d) How many conditions in time are needed to solve this equation?

7. Given the data below, what is the largest time step allowed in the simple explicit method of solving the Heat Equation when alpha = 0.5 cm^2/sec and $\Delta x = \Delta y = \Delta z = 5$ cm in
   d) One dimension
   e) Two dimensions
   f) Three dimensions