

Numerical Integration

Final Exam 1999S

2. Find the value of $\int_0^{0.6} f(x)dx$ by any **two** methods. Clearly identify the methods by name.

x	f(x)
0.0	2.00000
0.1	1.91984
0.2	1.67808
0.3	1.27352
0.4	0.70976
0.5	0.00000
0.6	-0.82816
0.7	-1.72792
0.8	-2.62848
0.9	-3.43024
1.0	-4.00000

Final Exam 2001F

3. Find the integral for $f(x)dx$ from $x = 0$ to 16 using Simpson's 1/3 Rule.

x	0	2	4	6	8	10	12	14	16
f(x)	30	12	26	56	64	50	45	44	48

Final Exam 2001F

3. Find the integral for $f(x)dx$ from $x = 0$ to 16 using Simpson's 1/3 Rule.

x	0	2	4	6	8	10	12	14	16
f(x)	30	12	26	56	64	50	45	44	48

Final Exam 2002F

7. Describe the purpose of the Gauss-Quadrature Method and briefly how is it used?
2. Show how to use Gaussian Quadrature to determine the value of the following integrals. Be specific.

a) $\int_{-1}^1 (2 - 3x^2 + 9x^4)dx$

b) $\int_2^6 (2 + 3\ln(x) - x^2)dx$

3. Find the integral for $f(x)dx$ from $x = 0$ to 1.6 using Simpson's 1/3 Rule.

x	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6
f(x)	3	1	-2	-5	2	7	9	10	9

Final Exam 2005S

10. Find the integral for $f(x)dx$ from $x = 2$ to 26 using Simpson's 1/3 Rule.

x	2	5	8	11	14	17	20	23	26
f(x)	5	1	-3	-5	2	7	9	10	20

Hour Exam 3 2005F

2. Show how to use Gaussian Quadrature to determine the value of the following integrals. Be specific.

a) $\int_{-1}^1 (5x \ln(4+x) - x^2) dx$

$$5\left(-\frac{1}{\sqrt{3}}\right)\ln\left(4-\frac{1}{\sqrt{3}}\right) - \left(-\frac{1}{\sqrt{3}}\right)^2 + 5\left(\frac{1}{\sqrt{3}}\right)\ln\left(4+\frac{1}{\sqrt{3}}\right) - \left(\frac{1}{\sqrt{3}}\right)^2$$

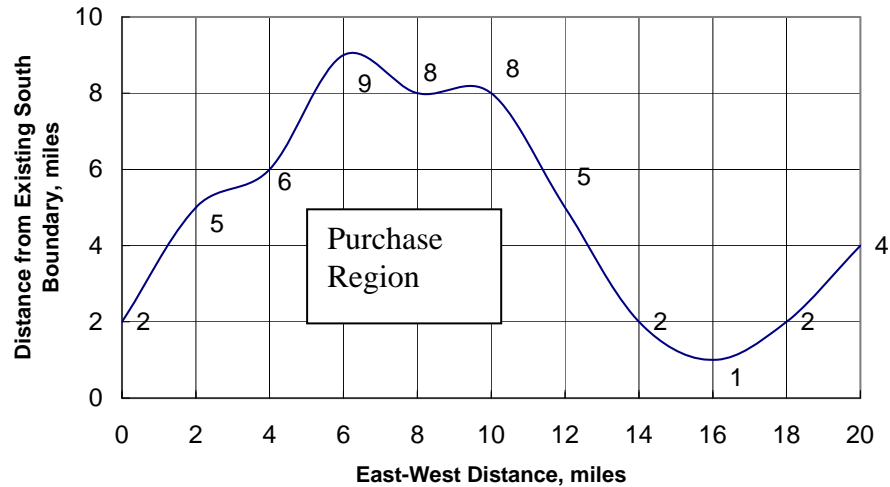
b) $\int_0^2 (5x \ln(x) - x^2) dx$

$$\left[5\left(1-\frac{1}{\sqrt{3}}\right)\ln\left(1-\frac{1}{\sqrt{3}}\right) - \left(1-\frac{1}{\sqrt{3}}\right)^2 + 5\left(1+\frac{1}{\sqrt{3}}\right)\ln\left(1+\frac{1}{\sqrt{3}}\right) - \left(1+\frac{1}{\sqrt{3}}\right)^2 \right]$$

c) $\int_0^{20} (5x \ln(x) - x^2) dx$

$$\left[5\left(10-\frac{10}{\sqrt{3}}\right)\ln\left(10-\frac{10}{\sqrt{3}}\right) - \left(10-\frac{10}{\sqrt{3}}\right)^2 + 5\left(10+\frac{10}{\sqrt{3}}\right)\ln\left(10+\frac{10}{\sqrt{3}}\right) - \left(10+\frac{10}{\sqrt{3}}\right)^2 \right] 10$$

5. An interstate highway is slated to cross the property of Big Land Holdings Corporation as shown below. The state has agreed to purchase the property south of the highway. Estimate the area to be purchased in square miles by each of the following methods:



Interstate 02

- a) Rectilinear Rule

$$I \approx h \sum_{i=0}^{n-1} x_i = 2[2+5+6+9+8+8+5+2+1+2] = \mathbf{92}$$

- b) Trapezoid Rule

$$I \approx h \sum_{i=0}^{n-1} \frac{(x_i + x_{i+1})}{2} = 2 \frac{[2+2*(5+6+9+8+8+5+2+1+2)+4]}{2} = \mathbf{96}$$

- c) Simpson's 1/3 Rule

$$I \approx \frac{h}{3} \sum_{i=0,2,\dots}^{n-2} (x_i + 4x_{i+1} + x_{i+2}) = \frac{2}{3} [2+4*(5+9+8+2+2)+2*(6+8+5+1)+4] = \mathbf{100}$$

Final Exam 2005F

8. Show how to use Gaussian Quadrature to determine the value of the following integrals. Be specific.

a) $\int_{-1}^1 (3x + 5x^2 - 9x^3) dx$

$$I = 3\left(\frac{-1}{\sqrt{3}}\right) + 5\left(\frac{-1}{\sqrt{3}}\right)^2 - 9\left(\frac{-1}{\sqrt{3}}\right)^3 + 3\left(\frac{1}{\sqrt{3}}\right) + 5\left(\frac{1}{\sqrt{3}}\right)^2 - 9\left(\frac{1}{\sqrt{3}}\right)^3 = \frac{5}{3}$$

$$b) \int_0^{20} (3\ln(x+1) + 5\sin(x^2) - 9x^3) dx$$

$$I = 3\ln\left(\left(10 - \frac{10}{\sqrt{3}}\right) + 1\right) + 5\sin\left(10 - \frac{10}{\sqrt{3}}\right)^2 - 9\left(10 - \frac{10}{\sqrt{3}}\right)^3 +$$

$$3\ln\left(\left(10 + \frac{10}{\sqrt{3}}\right) + 1\right) + 5\sin\left(10 + \frac{10}{\sqrt{3}}\right)^2 - 9\left(10 + \frac{10}{\sqrt{3}}\right)^3$$

11. Find the integral for $f(x)dx$ from $x = 1.0$ to 1.6 using Simpson's 1/3 Rule.

x	1.0	1.1	1.2	1.3	1.4	1.5	1.6
f(x)	9	12	20	16	8	5	4

$$I = \frac{0.1}{3}(9 + 4*12 + 2*20 + 4*16 + 2*8 + 4*5 + 4)$$