ODEs

Final Exam 1999S

3. The rate of change of z with t is given below. At t=1 , z = 3. Find z when t= 2 by any order Runge-Kutta method. Use a step size of 1.

$$\frac{dz}{dt} = (1+t) - 0.05z^2$$

Final Exam 2001F

4. The rate of change of y with t is given below and that at t = 0, y = 300. Describe how to find y over the range 0<t<20 by Runge-Kutta 4th Order. You may use an algebraic description, MathCad, MatLab, or any RK Solver but do identify your work.

$$\frac{dy}{dt} = -10 + 0.2t - .01y^2$$

Hour Exam #2 2005F

6. The following system of ODE's is to be solved with step size of 0.01 to t = 3.

$$\frac{dx}{dt} = f_x(t,x,y) = 0.2t + 2x/y^2$$

$$\frac{dy}{dt} = f_x(t,x,y) = 1 + 0.3\ln(xy)$$

$$t = 0 \quad x_0 = 2.1 \quad y_0 = 1$$

Choose one: (If you do both, only the first one will be graded.)

a) Write out the 4th order Runge-Kutta equations to show how to proceed through the solution.

b) Write out what the MathCad Solution looks like.

Solution: a) h = Δt = 0.01

$$x_{i+1} = x_i + \tfrac{1}{6}\left(k_{1x} + 2k_{2x} + 2k_{3x} + k_{4x}\right)$$

$$y_{i+1} = y_i + \tfrac{1}{6}\left(k_{1y} + 2k_{2y} + 2k_{3y} + k_{4y}\right)$$

where

$$k_{1x} = f_x(t,x,y)\Delta t \qquad k_{1y} = f_y(t,x,y)\Delta t$$

$$k_{2x} = f_x\left(t+\frac{\Delta t}{2}, x+\frac{k_{1x}}{2}, y+\frac{k_{1y}}{2}\right)\Delta t \qquad k_{2y} = f_y\left(t+\frac{\Delta t}{2}, x+\frac{k_{1x}}{2}, y+\frac{k_{1y}}{2}\right)\Delta t$$

$$k_{3x} = f_x\left(t+\frac{\Delta t}{2}, x+\frac{k_{2x}}{2}, y+\frac{k_{2y}}{2}\right)\Delta t \qquad k_{3y} = f_y\left(t+\frac{\Delta t}{2}, x+\frac{k_{2x}}{2}, y+\frac{k_{2y}}{2}\right)\Delta t$$

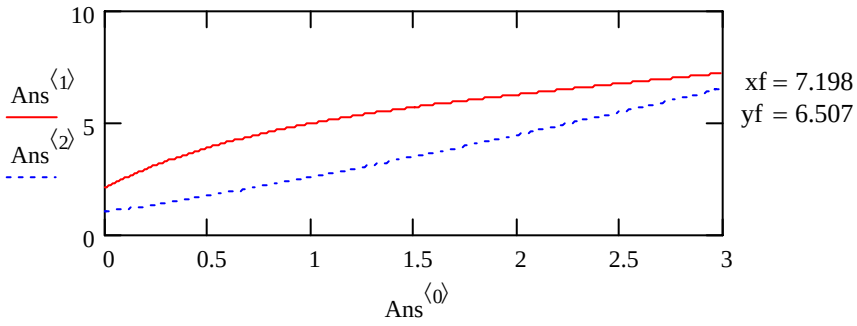
$$k_{4x} = fx(t+\Delta t, x+k_{3x}, y+k_{3y})\Delta t \qquad k_{4y} = f_y(t+\Delta t, x+k_{3x}, y+k_{3y})\Delta t$$

Solution: b) *(Actual MathCad Sheet)*

$fx(t,x,y) := 0.2 \cdot t + 2 \cdot \frac{x}{y^2}$ $\quad fy(t,x,y) := 1 + 0.3 \cdot \ln(x \cdot y)$

$I := \begin{pmatrix} 2.1 \\ 1 \end{pmatrix}$ $\quad D(t,z) := \begin{pmatrix} fx(t, z_0, z_1) \\ fy(t, z_0, z_1) \end{pmatrix}$ $\quad Ans := rkfixed(I, 0, 3, 300, D)$

$xf := Ans_{300,1}$ $\quad yf := Ans_{300,2}$



Final Exam 2005F

6. The rate of change of v with t is given below and that at t = 0, v = 300 find v when t = 0.2 by Runge-Kutta 4th Order using a step size of 0.2

$$\frac{dv}{dt} = -0.002\,v^2 + 0.35t$$