South Dakota School of Mines and Technology
Department of Computer and Mathematical Sciences

Math 373 Final Exam Dec 17, 2010

Turn in ONLY the printed sheets. Enter solutions in space provided ONLY.
There may be MORE data provided than needed to solve a problem.
No calculators, notes, books, reference materials

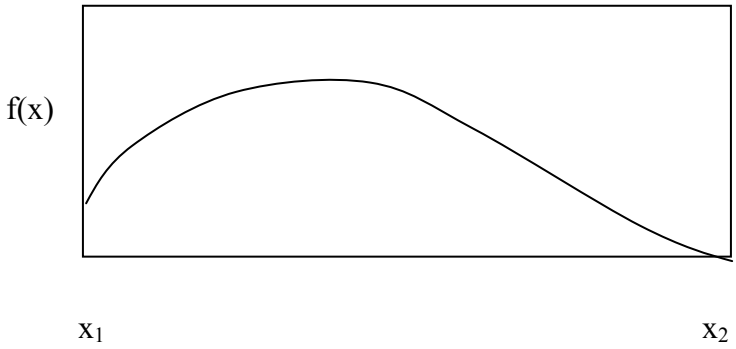
1. a) Write the 2D USS Heat Equation in incremental form.

$$\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)=\frac{\partial T}{\partial t}$$

b) Solve it for the new temperature at any time step.

c) Show the solution for the maximum time step.

2. Mark the location of ξ according to the Mean Value Theorem of Derivatives



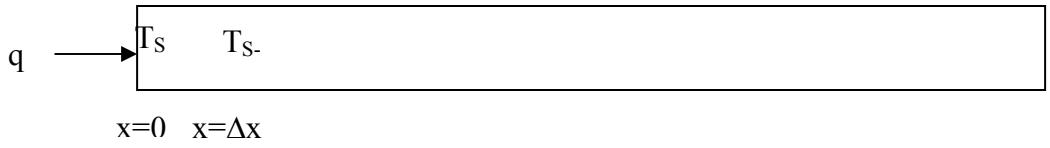
3 For $f(x) = 2x^3 + 3x^2 - 5$

a) Write the first order Taylor Series approximation in terms of x and h for the above function.

b) What is the value of ξ that makes the first order approximation exact when x = 1 and h=0.5?

4. Given the data below, what is the largest time step allowed in the method of solving a 1D USS HT problem by the methods covered so far in class if $\alpha = 0.5$ cm^2/sec and $\Delta x = 0.2$ cm?

5. A fixed heat flux of 10 Joules/(sec*cm^2) is being added into the solid steel bar below at x=0.



   a) Will the temperature at $x = \Delta x$ be lower, same, or higher than the temperature at $x = 0$ if $q>0$?

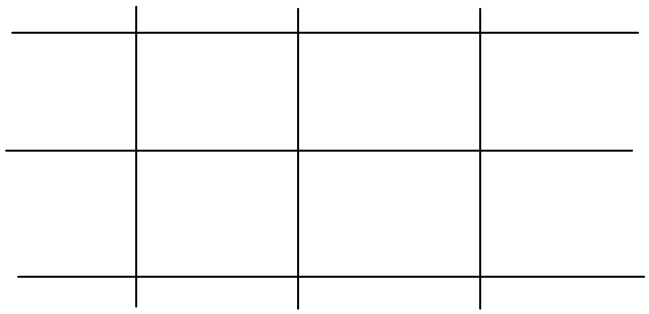
   b) Write the equation that describes the boundary temperature's relationship to the temperature at $x = \Delta x$. Use the notation in the sketch above.

6. Which of the following methods are explicit? Clearly cross out the ones that are not, if any.
   a) Saul'yev
   b) Dufort-Frankel
   c) Crank-Nicholson

7. a) How much energy can be stored at a surface with a boundary condition?

   b) Can the temperature change at a surface where a boundary conditions applies?

8. Describe the Dufort-Frankel Method. Be specific: equations, labeled sketch, values in equations, etc. Discuss skew, if any, in the approximations.



9. Using the data in Table 1 answer the following questions:
   a) What order polynomial do the data appear to observe?

   b) Approximate f(2.33) using a third order approximation. (Perform no arithmetic.)

Table 1. Difference Table for Interpolation

x	f(x)					
2.0	1.4000					
		0.4492				
2.1	1.8492		0.1518			
		0.6010		0.0206		
2.2	2.4502		0.1724		0.0010	
		0.7734		0.0216		
2.3	3.2236		0.1940		0.0010	
		0.9674		0.0226		
2.4	4.1910		0.2166		0.0010	
		1.1840		0.0235		
2.5	5.3750		0.2401		0.0010	
		1.4240		0.0245		
2.6	6.7990		0.2646		0.0010	
		1.6886		0.0254		
2.7	8.4876		0.2900			
		1.9786				
2.8	10.4662					

10. Show how to use Guassian Quadrature to determine the value of the following integrals. Be specific.

a) $\int_{-1}^{1}(2-3x^2+9x^4)dx$

b) $\int_{2}^{6}(2+3\ln(x)-x^2)dx$

11. Find the integral for f(x)dx from x = 0 to 1.6 using Simpson's 1/3 Rule.

x	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6
f(x)	3	1	-2	-5	2	7	9	10	9

12. Short Answer:
    a) What is the purpose of Data Adjustment?

    b) What is the mathematical basis (i.e. – the objective) of the method?

13. Layout the solution to the following set of equations using the Gauss-Seidel Method:

$$2x + y - z = 7$$
$$3x + 2y - 4z = 9$$
$$x - 7y - z = -12$$

14. Below are several LP tableaus in various states of completion. Describe the next step for each. Circle the pivot in any of the above tableaus that are ready to use a pivot.

a)

x	**y**	**z**	**S1**	**S2**	**S3**		**F**	**RHS**
5	3	2	1	0	0		0	1000
2	2	1	0	1	0		0	200
0	5	6	0	0	1		0	100
-60	-50	-2	0	0	0		1	0

b)

x	**y**	**z**	**S1**	**S2**	**S3**	**A**	**F**	**RHS**
5	4	2	1	0	0	0	0	1000
2	10	1	0	1	0	0	0	200
0	2	6	0	0	-1	1	0	100
-40	-50	-2	0	0	0	M	1	0

c)

x	**y**	**z**	**S1**	**S2**	**S3**	**A**	**F**	**RHS**
0	3	0	1	4	3	2	0	280
0	2	1	0	1	0	-5	0	52
1	5	0	0	0	1	3	0	21
0	50	0	0	35	5	14	1	4560