South Dakota School of Mines and Technology
Department of Computer and Mathematical Sciences

Math 373 **Final Exam** **Dec 18, 2001**

1. Incremenation of partial differentials:
   a) Write the following partial derivative in incremental form and state on the line provided whether your response is a backward, central, or forward approximation.

   $$\frac{\partial T}{\partial t} =$$

   Backward $\frac{T_t - T_{t-\Delta t}}{\Delta t}$; Central $\frac{T_{t+\Delta t} - T_{t-\Delta t}}{2\Delta t}$; Forward $\frac{T_{t+\Delta t} - T_t}{\Delta t}$

   b) Provide an explicit solution for the new T resulting from putting the Heat Equation into incremental form.

   $$\alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) = \frac{\partial T}{\partial t}$$

   $$T' = T + \frac{\alpha \Delta t}{\Delta x^2}\left[T_{+x} + T_{-x} + T_{+y} + T_{-y} - 4T\right]$$

2. Find a root of the following equation by any **two** numerical methods of your choice. To receive full credit you must layout the mathematical method clearly.

   $$x^3 - 2\ln(x) = 34$$

   $f(x) = x^3 - 2\ln(x) - 34$

   Newton's Method:

   $x_{N+1} = x_N - f(x)/f'(x) = (x_N{}^3 - 2\ln(x_N) - 34)/(3x_N{}^2 - 2/x_N)$

   One-Point Iteration:

   $x_{N+1} = [34 + 2\ln(x_N)]^{1/3}$

3. Find the integral for f(x)dx from x = 0 to 16 using Simpson's 1/3 Rule.

x	0	2	4	6	8	10	12	14	16
f(x)	30	12	26	56	64	50	45	44	48

$$I = \frac{h}{3}(f(x) + 4f(x) + f(x)) \quad h = 2$$

$$I = \frac{2}{3}(30 + 4(12) + 2(26) + 4(56) + 2(64) + 4(50) + 2(45) + 4(44) + 48)$$

$$I = 664$$

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4. Use the data below to estimate the value of f(6.6) using third order approximation (i.e. - cubic polynomial).

x	0	2	4	6	8	10	12	14	16
f(x)	30	12	26	56	64	50	45	44	48

Lagrangian Method

$$f(6.6) \quad = 56 * L_0(x) + 64 * L_1(x) + 50 * L_2(x) + 45 * L_3(x)$$

$$L_0(x) = \frac{(6.6-8)(6.6-10)(6.6-12)}{(6-8)(6-10)(6-12)}$$

$$L_1(x) = \frac{(6.6-6)(6.6-10)(6.6-12)}{(8-1.0)(8-2.0)(8-2.5)}$$

$$L_2(x) = \frac{(6.6-6)(6.6-8)(6.6-12)}{(10-6)(10-8)(10-12)}$$

$$L_3(x) = \frac{(6.6-6)(6.6-8)(6.6-10)}{(12-6)(12-8)(12-10)}$$

Newton's Difference Table

	0th order	1st order	2nd order	3rd order	4th order	etc
x	**f(x)**	**Δf[x]**	**Δ^2f[x]**	**Δ^3[x]**	**Δ^4f[x]**	
6.0	56					
		8				
8.0	64		-22			
		-14		31		
10.0	50		9		-36	
		-5		-5		-11
12.0	45		4		-47	
		-1		-52		
14.0	44		-48			
		-49.00				
3.5	-5					

$$f(x) = f(x_0) + \frac{\Delta f(x_0)}{h}(x - x_0) + \frac{\Delta^2 f(x_0)}{2!h^2}(x - x_0)(x - x_1) + \frac{\Delta^3 f(x_0)}{3!h^3}(x - x_0)(x - x_1)(x - x_2)$$

$$= 56 + \frac{64}{h}(6.6-6) - \frac{50}{2!h^2}(6.6-6)(6.6-8) - \frac{45}{3!h^3}(6.6-6)(6.6-8)(6.6-10)$$

where $h = 2$

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5. The rate of change of v with t is given below and that at t = 0, v = 300 find v when t = 0.2 by Runge-Kutta 4th Order using a step size of 0.2

$$\frac{dv}{dt} = -10 + t/2$$

Five pieces of information are needed:

$$^{(1)}F(t,v) = -10 + \frac{t}{2}$$

$$^{(2)}t = 0$$

$$^{(3)}v_0 = 300$$

$$^{(4)}t = 0.2$$

$$^{(5)}\Delta t = 0.2$$

$$F(t,y) \quad I := (300) \quad D(t,Q) := (F)$$

$$Answer := RKfixed(I, 0, 0.2, 0.2, D)$$

6. Given the data below, what is the largest time step allowed in the simple explicit method of solving the Heat Equation when alpha = 0.25 cm^2/sec and Δx = Δy = Δz = 0.1 cm in
   a) One dimension
      1/2
   b) Two dimensions
      1/4
   c) Three dimensions
      1/6

7. Describe the purpose of the Guass-Quadrature Method and briefly how is it used?
A numerical integration of third order accuracy that consists of simply adding the function at two specified locations $\pm(1/3)^{1/3}$ for integration over the interval -1 to +1.

8. Show how to solve the following set of linear equations using the Gauss-Seidel method

$$7x^3 + 3y + 5z = 14$$

$$2x + y^2 = 6$$

$$x^2 + 2y + 5z^3 = 18$$

Make the diagonal dominant and solve each of the equations for each variable as follows:

$$x = \sqrt[3]{\frac{14-3y-5z}{7}}$$

$$y = \sqrt{6-2x}$$

$$z = \sqrt[3]{\frac{18-x^2-2y}{5}}$$

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9. The solid bar below is conducting heat along its horizontal axis while heat is being lost by convection from both ends. Derive the boundary equations showing the temperature at the rod ends as a function of external fluid temperature T_a and the temperatures $T_{\Delta x}$ and $T_{L-\Delta x}$ each one increment in from each end. (Assume there is no heat loss from the surface perpendicular to the horizontal axis of the bar.)

Note: $q_{conv} = h(T-T_a)$
$q_{cond} = -k(dT/dx)$

$$q_{conv} = q_{cond}$$

$$-k\frac{T_s - T_{s-}}{\Delta x} = h(T_s - T_a)$$

$$T_s = \left(\frac{T_a h + T_s \frac{k}{\Delta x}}{\left(h + \frac{k}{\Delta x} \right)} \right)$$