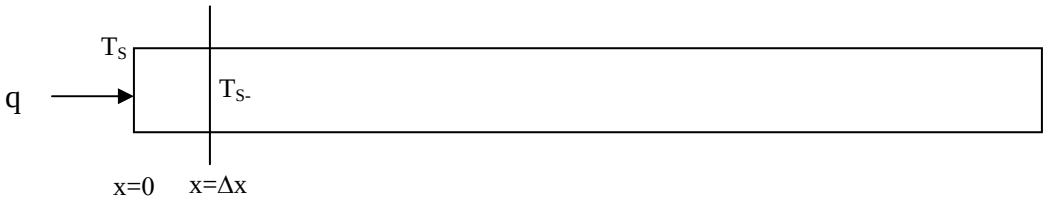
Name:_____

# South Dakota School of Mines and Technology
# Mathematical Sciences Department

Math 373 **HQ 2 Turn in ONLY the printed sheets** Nov 5, 2001

**Discard scratch paper. Put your answers in the space provided on each sheet.**
**Weight: Name 4 points, problems 1- 6 are worth 16 points each. Total = 100**

1. A fixed heat flux of 10 Joules/(sec*cm$^2$) is being added into the solid steel bar below at x=0.




   a) Will the temperature at x = Δx be lower, same, or higher than the temperature at x = 0?

   b) Write the equation that describes the boundary temperature's relationship to the temperature at x = Δx. Use the notation in the sketch above.

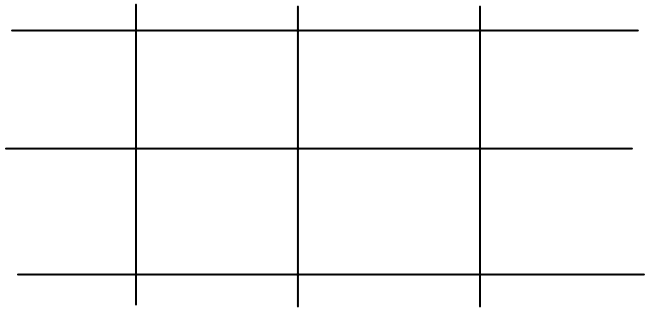
2. Which of the following methods are explicit? Clearly cross out the ones that are not, if any.
   a) ADI
   b) Saul'yev
   c) Dufort-Frankel
   d) Crank-Nicholson

3. a) How much energy can be stored at a surface with a boundary condition?

   b) Can the temperature change at a surface where a boundary conditions applies?

4. What happened to the "a" term in the first equation and the "c" term in the last equation of the tridiagonal matrix obtained in the implicit methods for determining temperature profiles in solids?

5. Describe the Dufort-Frankel Method. Be specific: equations, labeled sketch, values in equations, etc. Discuss skew, if any, in the approximations.



6. Given the Tridiagonal Matrix Algorithm below, find three errors in the following macro for the algorithm: Sub Tridiag(n, d, Tnew) **Every wrong answer will void one correct answer so be careful.**

```
' n = number of unknown T's
' d = RHS matrix

Dim a, b, c, d(10), Tnew(10), T(10)
.
.
.
```

**Clearly write the correct code below**

```
Sub Tridiag(n, d, Tnew)
Dim beta(100), gamma(100)
beta(1) = b
gamma(1) = b(1) / beta(1)
For i = 2 To n-1
beta(i) = b - a * c / beta(i)
gamma(i) = d(i) – (a * gamma(i - 1)) / beta(i)
Next I
Tnew(n) = gamma(n)
For i = n - 1 To 1 Step –1
Tnew(i) = gamma(i) - c * T(i + 1) / beta(i)
Next n
End Function
```

**Tridiagonal Matrix Algorithm**

Crank-Nicholson has all the same a's, b's, and c's and are not subscripted. The general Tridagonal Algorithm allows unique values of a, b, and c in each equation and are, therefore, subscripted.

$$T_N = \gamma_N \tag{17}$$

$$T_i = \gamma_i - \frac{c_i T_{i+1}}{\beta_i} \qquad i = N-1,\ N-2,\ ....1 \tag{18}$$

Where the β's and γ's are determined from the recurrision formulas

$$\beta_1 = b_1, \qquad \gamma_1 = d_1/\beta_1,$$

$$\beta_i = b_i - \frac{a_i c_{i-1}}{\beta_{i-1}} \qquad i = 2,3,\ .....,\ N \tag{19a, b, c, d}$$

$$\gamma_i = \frac{d_i - a_i \gamma_{i-1}}{\beta_i} \qquad i = 2,3,\ .....,\ N.$$

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# Detach this sheet and turn it in Wednesday, Nov 7 at 2:50 PM

(Answers provide - for the observant listener - in lecture on Wednesday)

7. The following question may be turned at the end of class Wednesday, Nov 7, for up to **eight** replacement points on this exam. The lecture will include the answers. No late papers except for bona fide absences as determined by Dr. Howard in which case some alternative will be made available.

   ***Dynamic programming problems have eight common characteristics as identified by our guest lecture, Dr. Kellogg. Cite four of these characteristics. Two points per correct answer up to a maximum of eight points total.***

   ***1.***

   ***2.***

   ***3.***

   ***4.***