South Dakota School of Mines and Technology
Department of Computer and Mathematical Sciences

Name: ___________________
(printed neatly)

Math 373 **HQ 2 (Closed notes, books, calculators)** **Nov 1, 2005**

Turn in ONLY the printed sheets with your solutions in space provided.

If a question seems to contain an error, state the trouble and a reasonable correction and proceed with the solution under your assumed correction.

1. Layout the solution to the following set of equations using the Gauss-Seidel Method:

$$2x + y - 5z = -3$$
$$3x + 3y - 4z = -1$$
$$9x - 2y - z = -30$$

Solution:

Make a dominant Diagonal:

$$9x - 2y - z = -30$$
$$3x + 3y - 4z = -1$$
$$2x + y - 5z = -3$$

Solve for each variable in turn:

$$x = (-30 + 2y + z)/9$$
$$y = (-1 - 3x + 4z)/3$$
$$z = (-3 - 2x - y)/(-5)$$

Start computational Sequence: Assume: $y_0 = z_0 = 1$

N	x	y	z
0	[redacted]	1	1
1	-3.00000	4.00000	0.20000
2	-2.42222	2.35556	0.10222
3	-2.79852	2.60148	0.00089
4	-2.75513	2.42298	-0.01746
5	-2.79683	2.44023	-0.03069
6	-2.79447	2.42022	-0.03374
7	-2.79926	2.42093	-0.03552
8	-2.79930	2.41861	-0.03600
9	-2.79986	2.41854	-0.03624
10	-2.79991	2.41826	-0.03631
11	-2.79998	2.41823	-0.03635

2. Which of the below methods could be used to numerically solve a 1D USS HC problem for a time step of 10 seconds if $\alpha = 0.5\ cm^2/sec$ and $\Delta x = 2$ cm?
   a) Elementary Explicit _No_____ If "No", why? Max $\Delta t = 0.5* \Delta x^2/ \alpha = 4$ sec____________
   b) Saul'yev _Yes_____________ If "No", why? ____
   c) Crank-Nicolson _Yes_________ If "No", why? ____
   d) DuFort-Frankel _Yes_________ If "No", why? ____

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3. Complete the general Tridiagonal Matrix Algorithm macro below that will be sent N and the arrays a, b, c, and d. It will return the solved values of T. **There may be more lines than needed. Correct anything that is wrong.**

   ***Solution*:** Underlined Text

```
Sub TriDiag(N, a, b, c, d, T )
    Dim a(100), b(100), c(100), d(100), T(100), Beta(100), Gamma(100)
    Beta(1)=b(1)
    Gamma(1)=d(1)/Beta(1)
    For k= 1 to N
        Beta(k) = b(k) -a(k)c(k-1)/Beta(k-1)
        Gamma(k) = (d(k) -a(k)Gamma(k-1)/Beta(k)
    Next N
    T(N) = Gamma(N)
    For k= N-1 to 1 step -1
        T(k) = Gamma(k)-c(k)*T(k+1)/Beta(k)
    Next N
End Sub
```

Tridiagonal Matrix Algorithm

$$T_N = \gamma_N$$

$$T_i = \gamma_i - \frac{c_i T_{i+1}}{\beta_i} \qquad i = N-1,\ N-2,\ \ldots.1$$

Where the β's and γ's are determined from the recurrision formulas

$$\beta_1 = b_1, \qquad \gamma_1 = d_1 / \beta_1,$$

$$\beta_i = b_i - \frac{a_i c_{i-1}}{\beta_{i-1}} \qquad i = 2, 3,\ \ldots\ldots,\ N$$

$$\gamma_i = \frac{d_i - a_i \gamma_{i-1}}{\beta_i} \qquad i = 2, 3,\ \ldots\ldots,\ N.$$

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4 Setup the one-point iteration to find the root to the following equations starting with x = 1.

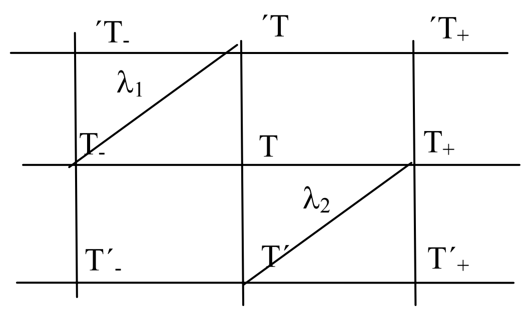
$$x^{2.6} - 2\ln(x) - x = 15$$

Solution:

$X_{N+1} = [15 + X_N + 2*\ln(X_N)]^{(1/2.6)} = g(X)$. Start with $X_1 = 1$

N	X_N	g(x)
1	1	2.9048
2	2.90485	3.1675
3	3.16745	3.1938
4	3.19377	3.1963
5	3.19634	3.1966
6	3.19659	3.1966

5. Describe the following "fast" methods for solving a 1D USS HT problem using
   a) DuFort-Frankel Method. Use the grid and label everything needed in the equations written.
   Solution:



$$\alpha \frac{\partial^2 T}{\partial x^2} = \frac{\delta T}{\delta t}$$

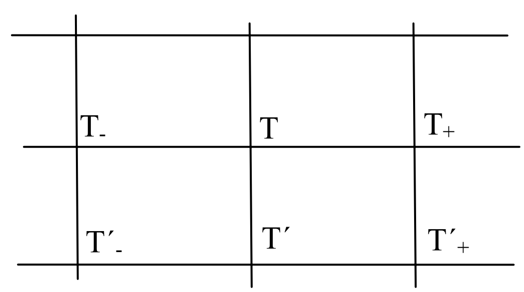
$$\alpha \frac{\lambda_2 - \lambda_1}{\Delta x} = \frac{T' - \acute{T}}{2\Delta t}$$

where

$$\lambda_2 = (T_+ - T')/\Delta x \qquad \lambda_1 = (\acute{T} - T_-)/\Delta x$$

Solve for T' (explicit)

b) Saul'yev
   Solution:



$$\alpha \frac{\partial^2 T}{\partial x^2} = \frac{\delta T}{\delta t}$$

$$\alpha \frac{\lambda_2 - \lambda_1}{\Delta x} = \frac{T' - T}{2\Delta t}$$

where:

$$left-to-right: \lambda_2 = (T_+ - T)/\Delta x \qquad \lambda_1 = (T' - T'_-)/\Delta x$$

$$right-to-left: \lambda_2 = (T'_+ - T')/\Delta x \qquad \lambda_1 = (T - T_-)/\Delta x$$

Solve for T' (explicit)

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6. The following system of ODE's is to be solved with step size of 0.01 to t = 3.

$$\frac{dx}{dt} = f_x(t,x,y) = 0.2t + 2x/y^2$$

$$\frac{dy}{dt} = f_x(t,x,y) = 1 + 0.3\ln(xy)$$

$$t = 0 \qquad x_0 = 2.1 \qquad y_0 = 1$$

Choose one: (If you do both, only the first one will be graded.)
a) Write out the 4th order Runge-Kutta equations to show how to proceed through the solution.
b) Write out what the MathCad Solution looks like.

Solution: a) h = Δt = 0.01

$$x_{i+1} = x_i + \tfrac{1}{6}\left(k_{1x} + 2k_{2x} + 2k_{3x} + k_{4x}\right)$$

$$y_{i+1} = y_i + \tfrac{1}{6}\left(k_{1y} + 2k_{2y} + 2k_{3y} + k_{4y}\right)$$

where

$$k_{1x} = f_x(t,x,y)\Delta t \qquad k_{1y} = f_y(t,x,y)\Delta t$$

$$k_{2x} = f_x(t+\frac{\Delta t}{2}, x+\frac{k_{1x}}{2}, y+\frac{k_{1y}}{2})\Delta t \qquad k_{2y} = f_y(t+\frac{\Delta t}{2}, x+\frac{k_{1x}}{2}, y+\frac{k_{1y}}{2})\Delta t$$

$$k_{3x} = f_x(t+\frac{\Delta t}{2}, x+\frac{k_{2x}}{2}, y+\frac{k_{2y}}{2})\Delta t \qquad k_{3y} = f_y(t+\frac{\Delta t}{2}, x+\frac{k_{2x}}{2}, y+\frac{k_{2y}}{2})\Delta t$$

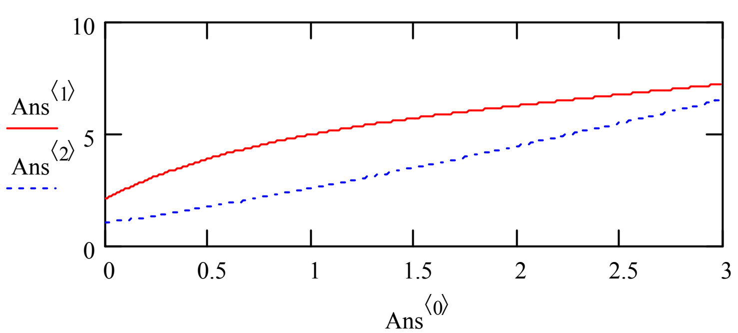
$$k_{4x} = fx(t+\Delta t, x+k_{3x}, y+k_{3y})\Delta t \qquad k_{4y} = f_y(t+\Delta t, x+k_{3x}, y+k_{3y})\Delta t$$

Solution: b) *(Actual MathCad Sheet)*

$$fx(t,x,y) := 0.2\cdot t + 2\cdot\frac{x}{y^2} \qquad fy(t,x,y) := 1 + 0.3\cdot\ln(x\cdot y)$$

$$I := \begin{pmatrix} 2.1 \\ 1 \end{pmatrix} \qquad D(t,z) := \begin{pmatrix} fx(t,z_0,z_1) \\ fy(t,z_0,z_1) \end{pmatrix} \qquad Ans := rkfixed(I,0,3,300,D)$$

$$xf := Ans_{300,1} \qquad yf := Ans_{300,2}$$



xf = 7.198
yf = 6.507