

**South Dakota School of Mines and Technology**  
**Department of Mathematical and Computer Sciences**

Math 373

HQ 3 Solution

Dec 2, 2005

If a question is in error or unclear, state a professional fix for it and proceed. Proctor will not answer any questions.

**No Calculators – Closed Book and Notes**

1. Below are some regularly-spaced data from a polynomial function.

$x$	$f(x)$	$Df(x)$	$D^2f(x)$	$D^3f(x)$
1	1			
		0		
2	1	8	8	6
3	9	22	14	6
4	31	42	20	6
5	73	68	26	6
6	141			

a) What is the order of the polynomial from which the data appear to have been generated? **Why?**

**Third Order since all differences above third order are zero.**

b) Reconstruct the polynomial function that fits the data exactly.

$$\begin{aligned} f(x) &= 1+0(x-1)+8(x-1)(x-2)/2+6(x-1)(x-2)(x-3)/6 \\ &= 1+4x^2-12x+8+x^3-3x^2+2x-3x^2+9x-6 \\ &= 3-x-2x^2+x^3 \end{aligned}$$

c) Show the best *first-order* approximation of  $f(3.1)$ .

**Use the closest value to 3.1 for the  $x_0$  (=3) term:  $f(3.1) = 9+22(3.1-3) = 11.2$**

2. Show how to use Gaussian Quadrature to determine the value of the following integrals. Be specific.

a)  $\int_{-1}^1 (5x \ln(x) - x^2) dx$

**Clearly one cannot evaluate a ln at negative values. If one could the answer would be**

$$5\left(-\frac{1}{\sqrt{3}}\right)\ln\left(-\frac{1}{\sqrt{3}}\right)-\left(-\frac{1}{\sqrt{3}}\right)^2 + 5\left(\frac{1}{\sqrt{3}}\right)\ln\left(\frac{1}{\sqrt{3}}\right)-\left(\frac{1}{\sqrt{3}}\right)^2$$

**The problem might be restated with a function  $5x \ln(10+x)-x^2$ . The each of the above ln terms would contain an additional value of 4.**

$$5\left(-\frac{1}{\sqrt{3}}\right)\ln\left(4-\frac{1}{\sqrt{3}}\right)-\left(-\frac{1}{\sqrt{3}}\right)^2 + 5\left(\frac{1}{\sqrt{3}}\right)\ln\left(4+\frac{1}{\sqrt{3}}\right)-\left(\frac{1}{\sqrt{3}}\right)^2$$

b)  $\int_0^2 (5x \ln(x) - x^2) dx$

$$\left[ 5\left(1-\frac{1}{\sqrt{3}}\right)\ln\left(1-\frac{1}{\sqrt{3}}\right)-\left(1-\frac{1}{\sqrt{3}}\right)^2 + 5\left(1+\frac{1}{\sqrt{3}}\right)\ln\left(1+\frac{1}{\sqrt{3}}\right)-\left(1+\frac{1}{\sqrt{3}}\right)^2 \right]$$

c)  $\int_0^{20} (5x \ln(x) - x^2) dx$

$$\left[ 5\left(10-\frac{10}{\sqrt{3}}\right)\ln\left(10-\frac{10}{\sqrt{3}}\right)-\left(10-\frac{10}{\sqrt{3}}\right)^2 + 5\left(10+\frac{10}{\sqrt{3}}\right)\ln\left(10+\frac{10}{\sqrt{3}}\right)-\left(10+\frac{10}{\sqrt{3}}\right)^2 \right] 10$$

3. Describe how to find the best values for the coefficients for the equation

$$f(x) = a + b \ln(x) + cx^3$$

given the following data:

x	f(x)	fcalc(x)	Error=(f(x)-fcalc(x)) <sup>2</sup>
1	2.20	$= a + b \ln(1) + c1^3$	$[2.20 - (a + b \ln(1) + c1^3)]^2$
1.2	2.91	$= a + b \ln(1.2) + c(1.2)^3$	etc
1.6	4.28	$= a + b \ln(1.6) + c(1.6)^3$	etc
2.2	6.57	$= a + b \ln(2.2) + c(2.2)^3$	etc
3.1	11.47	$= a + b \ln(3.1) + c(3.1)^3$	etc
6.3	57.72	$= a + b \ln(6.3) + c(6.3)^3$	etc
5.4	38.72	$= a + b \ln(5.4) + c(5.4)^3$	etc
			<b>Sum of Errors<sup>2</sup></b>

Search a, b, c space for MIN **Sum of Errors<sup>2</sup>** using Solver in Microsoft Excel<sup>®</sup>.

4. Below are several LP tableaus in various states of completion. Describe the next step for each.

a)

X1	X2	X3	X4	S1	S2	S3	A1	A2	F	RHS
1	3			1						280
	2	1	1		-1		1			52
	5					1				14
1	1	3	2					1		240
-9	-6	-1	-5				10	10	1	0
							0	0		

**Drive out the Big M's in Column A1 and A2**

b)


x	y	z	S1	S2	S3	F	RHS
1		2	12	3			20
	1	1	21	1			54
		6	0.6	5	1		100
		15	58	6		1	2365

**Completed**

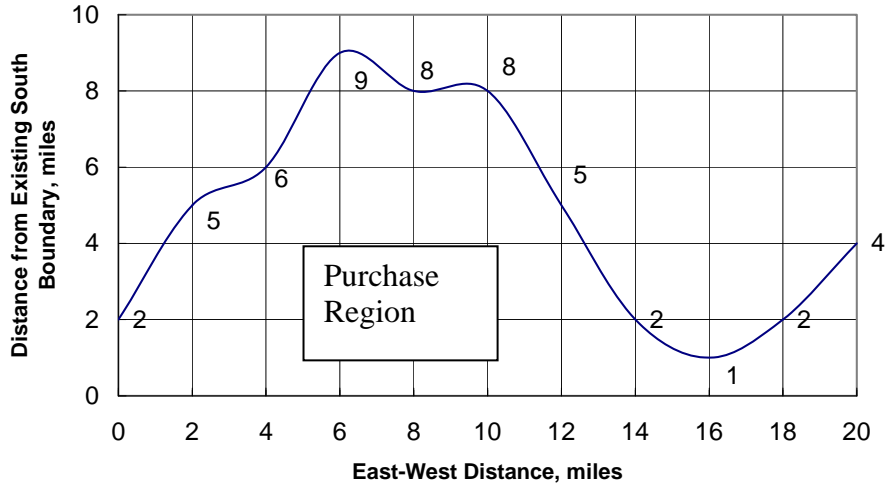
c)

X1	X2	X3	S1	S2	S3	F	RHS
7	4	2	1				2000
2	10	1		1			250
	2	6			1		160
-60	-50	-2				1	0

**Enter X1 into the BFS**

d) Circle the pivot(s) in any of the above tableaus that are ready to use a pivot. See 

5. An interstate highway is slated to cross the property of Big Land Holdings Corporation as shown below. The state has agreed to purchase the property south of the highway. Estimate the area to be purchased in square miles by each of the following methods:



**Interstate 02**

- a) Rectilinear Rule

$$I \approx h \sum_{i=0}^{n-1} x_i = 2[2+5+6+9+8+8+5+2+1+2] = \mathbf{96}$$

- b) Trapezoid Rule

$$I \approx h \sum_{i=0}^{n-1} \frac{(x_i + x_{i+1})}{2} = 2 \frac{[2+2*(5+6+9+8+8+5+2+1+2)+4]}{2} = \mathbf{98}$$

- c) Simpson's 1/3 Rule

$$I \approx \frac{h}{3} \sum_{i=0,2,\dots}^{n-2=8} (x_i + 4x_{i+1} + x_{i+2}) = \frac{2}{3} [2+4*(5+9+8+2+2)+2*(6+8+5+1)+4] = \mathbf{100}$$